AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level MATHEMATICS

Unit Statistics 2B

Tuesday 21 June 2016 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions.

Answer each question in the space provided for that question.

1 The water in a pond contains three different species of a spherical green algae:

Volvox globator, at an average rate of 4.5 spheres per $1\,\text{cm}^3$;

Volvox aureus, at an average rate of 2.3 spheres per $1\,\text{cm}^3$;

Volvox tertius, at an average rate of 1.2 spheres per $1\,\text{cm}^3$.

Individual *Volvox* spheres may be considered to occur randomly and independently of all other *Volvox* spheres.

Random samples of water are collected from this pond.

Find the probability that:

(a) a $1\,\text{cm}^3$ sample contains no more than 5 *Volvox globator* spheres;

[1 mark]

(b) a $1\,\text{cm}^3$ sample contains at least 2 *Volvox aureus* spheres;

[3 marks]

(c) a $5\,\text{cm}^3$ sample contains more than 8 but fewer than 12 *Volvox tertius* spheres;

[3 marks]

(d) a $0.1\,\text{cm}^3$ sample contains a **total** of exactly 2 *Volvox* spheres;

[3 marks]

(e) a $1\,\text{cm}^3$ sample contains at least 1 sphere of each of the three different species of algae.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 1**

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 1**

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 1**

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 1**

Do not write outside the box

2 A normally distributed variable, X, has unknown mean μ and unknown standard deviation σ.

A sample of 10 values of X was taken. From these 10 values, a 95% confidence interval for μ was calculated to be

$$(30.47,\ 32.93)$$

Use this confidence interval to find unbiased estimates for μ and σ^2.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 2

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 2**

Do not write outside the box

3 Members of a library may borrow up to 6 books. Past experience has shown that the number of books borrowed, X, follows the distribution shown in the table.

x	0	1	2	3	4	5	6
$\mathbf{P}(X = x)$	0	0.19	0.26	0.20	0.13	0.07	0.15

(a) Find the probability that a member borrows more than 3 books.

[1 mark]

(b) Assume that the numbers of books borrowed by two particular members are independent.

Find the probability that one of these members borrows more than 3 books and the other borrows fewer than 3 books.

[3 marks]

(c) Show that the mean of X is 3.08, and calculate the variance of X.

[4 marks]

(d) One of the library staff notices that the values of the mean and the variance of X are similar and suggests that a Poisson distribution could be used to model X.

Without further calculations, give **two** reasons why a Poisson distribution would **not** be suitable to model X.

[2 marks]

(e) The library introduces a fee of 10 pence for each book borrowed.

Assuming that the probabilities do not change, calculate:

(i) the mean amount that will be paid by a member;

(ii) the standard deviation of the amount that will be paid by a member.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 3

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 3**

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 3**

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 3**

Do not write outside the box

4 A digital thermometer measures temperatures in degrees Celsius. The thermometer rounds down the actual temperature to one decimal place, so that, for example, 36.23 and 36.28 are both shown as 36.2. The error, $X\,°\text{C}$, resulting from this rounding down can be modelled by a rectangular distribution with the following probability density function.

$$\text{f}(x) = \begin{cases} k & 0 \leqslant x \leqslant 0.1 \\ 0 & \text{otherwise} \end{cases}$$

(a) State the value of k.

[1 mark]

(b) Find the probability that the error resulting from this rounding down is greater than $0.03\,°\text{C}$.

[1 mark]

(c) (i) State the value for $\text{E}(X)$.

(ii) Use integration to find the value for $\text{E}(X^2)$.

(iii) Hence find the value for the standard deviation of X.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 4

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 4**

Do not write outside the box

5 A car manufacturer keeps a record of how many of the new cars that it has sold experience mechanical problems during the first year. The manufacturer also records whether the cars have a petrol engine or a diesel engine. Data for a random sample of 250 cars are shown in the table.

	Problems during first 3 months	**Problems during first year but after first 3 months**	**No problems during first year**	**Total**
Petrol engine	10	35	170	215
Diesel engine	4	8	23	35
Total	14	43	193	250

(a) Use a χ^2-test to investigate, at the 10% significance level, whether there is an association between the mechanical problems experienced by a new car from this manufacturer and the type of engine.

[11 marks]

(b) Arisa is planning to buy a new car from this manufacturer. She would prefer to buy a car with a diesel engine, but a friend has told her that cars with diesel engines experience more mechanical problems.

Based on your answer to part **(a)**, state, with a reason, the advice that you would give to Arisa.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 5

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 5**

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 5**

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 5**

Do not write outside the box

6 Gerald is a scientist who studies sand lizards. He believes that sand lizards on islands are, on average, shorter than those on the mainland. The population of sand lizards on the mainland has a mean length of $18.2\,\text{cm}$ and a standard deviation of $1.8\,\text{cm}$.

Gerald visited three islands, A, B and C, and measured the length, X centimetres, of each of a sample of n sand lizards on each island. The samples may be regarded as random. The data are shown in the table.

Island	$\sum x$	n
A	1384.5	78
B	116.9	7
C	394.6	20

(a) Carry out a hypothesis test to investigate whether the data from Island A provide support for Gerald's belief at the 2% significance level. Assume that the standard deviation of the lengths of sand lizards on Island A is $1.8\,\text{cm}$.

[7 marks]

(b) For Island B, it is also given that

$$\sum (x - \bar{x})^2 = 22.64$$

(i) Construct a 95% confidence interval for μ_B, where μ_B centimetres is the mean length of sand lizards on Island B. Assume that the lengths of sand lizards on Island B are normally distributed with **unknown** standard deviation.

(ii) Comment on whether your confidence interval provides support for Gerald's belief.

[7 marks]

(c) Comment on whether the data from Island C provide support for Gerald's belief.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 6

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 6**

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 6**

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 6

Do not write outside the box

7 The continuous random variable X has a cumulative distribution function $\mathrm{F}(x)$, where

$$\mathrm{F}(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{4}(x-1) & 1 \leqslant x < 4 \\ \frac{1}{16}(12x - x^2 - 20) & 4 \leqslant x \leqslant 6 \\ 1 & x > 6 \end{cases}$$

(a) Sketch the probability density function, $\mathrm{f}(x)$, on the grid below.

[5 marks]

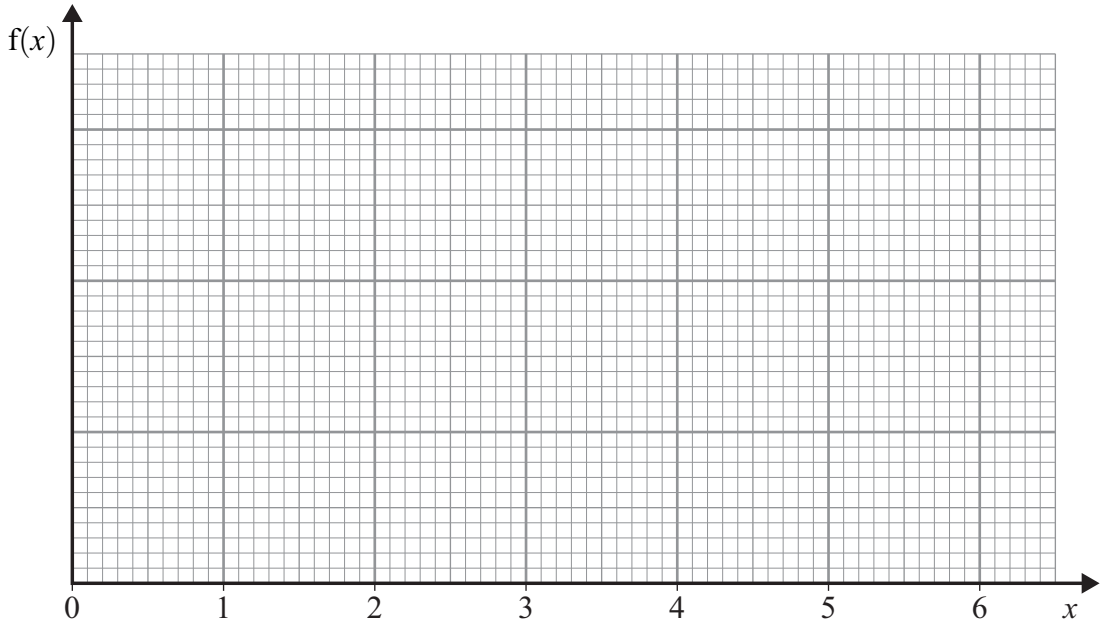
(b) Find the mean value of X.

[4 marks]

QUESTION PART REFERENCE

(a)

Answer space for question 7



Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 7**

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 7**

END OF QUESTIONS

Copyright information

For confidentiality purposes, from the November 2015 examination series, acknowledgements of third party copyright material will be published in a separate booklet rather than including them on the examination paper or support materials. This booklet is published after each examination series and is available for free download from www.aqa.org.uk after the live examination series.

Permission to reproduce all copyright material has been applied for. In some cases, efforts to contact copyright-holders may have been unsuccessful and AQA will be happy to rectify any omissions of acknowledgements. If you have any queries please contact the Copyright Team, AQA, Stag Hill House, Guildford, GU2 7XJ.

Copyright © 2016 AQA and its licensors. All rights reserved.